ON THE LIFETIME OF A COLD DARK MATTER PARTICLE AND THE COSMOLOGICAL DIFFUSE PHOTON BACKGROUND

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We show that a Majorana heavy neutrino with a mass $\mathcal{O}(10\ \mathrm{TeV})$ is a good candidate particle for cold dark matter. It can be responsible for the majority of the cosmological diffuse photon background owing to lifetime of the order of $\mathcal{O}(10^{19}s)$, dominantly fixed by the radiative two-body decay. The lifetime is suppressed by two mechanisms: the leptonic GIM cancellation and the see-saw weak coupling suppression. As a fermion cold dark matter particle, a heavy neutrino favours the average mass density of the Universe constrained by the Einstein-Cartan cosmology.

1 Introduction and motivation

During the past decades we have been witnessed of great progress in cosmology which places severe constraints on the physics of fundamental constituents of matter. There is a common belief that it is not possible to resolve the problems of structure formation of the Universe, basic cosmological parameters, nucleosynthesis or a diffuse photon background without the introduction of cold and hot dark matter and the violation of the baryon number.

On the other hand, measurements of the LSND and the SuperKamiokande have definitely confirmed the existence of massive neutrinos and their flavour mixing, urging our necessity to build a more predictable theory in particle physics than the Standard Model.

A diffuse photon background as a source of ionization at galactic or extragalactic scales represents a challenge for particle physics to search for a process that can explain the phenomenon [1]. It has been known for a long time that the measurement of the flux of decay-produced photons can constrain the lifetime of the decayed particle [2, 3, 1]. From a detailed study of ionization fluxes [1], Sciama has estimated that the mass and the lifetime of a decaying light neutrino are $m_{\nu_\tau} \simeq 28eV,\ \tau \simeq 10^{24}s$. However, it is very difficult to reconcile such a high mass with the present cosmological fits of the structure formation which require an order of magnitude smaller neutrino masses (as hot dark matter) and cold dark matter (that should dominate the mass density of the Universe). The observed neutrino oscillations at the SuperKamiokande and the limits from other detectors prohibit large masses if we exclude mass degeneracy. The standard weak interaction calculation of the neutrino lifetime[3] with a mass $m_\nu \simeq 30eV$ gives $\tau \simeq 10^{36}s$, thus twelve orders of magnitude larger than it is required by the ionization flux. One can improve the result adding a contribution of a new scalar or vector charged particle exchanged in the quantum loop, but new particles could completely spoil the structure of electroweak interactions that are verified to very high precision at LEP and SLD. To measure a photon flux of the decaying neutrino near the Sun, an extreme ultraviolet detector on the satellite has been proposed[1]. In 1997 the satellite was launched and now there are data that are incompatible with Sciama's decaying neutrino[4].

It seems that light neutrinos are not very promising candidates to solve the cosmological problem of diffuse photon background.

In this paper we investigate abundances and lifetimes of heavy neutrinos, within an electroweak theory proposed a few years ago, in order to find a cold dark matter(CDM) particle whose radiative decay could be responsible for a diffuse photon background.

2 Cosmic abundances of heavy neutrinos

To investigate the cosmological significance of any particle, one should know its interactions and cross sections to solve the Boltzmann equations in curved spacetime. Let us recall some results on the freeze-out and abundances of heavy neutrinos.

Heavy neutrinos with masses $\mathcal{O}(1GeV)$ can play a role of the CDM particle owing to high abundances calculated from the Z-boson mediated annihilation cross section [2] with the following scaling $\sigma_Z \propto m_N^2$. If neutrinos have masses much larger than weak bosons, the authors of Ref.[5] showed that the total annihilation cross section was not dominated by the fermion pair production $\sigma(\bar{N}N \to \bar{f}f) \propto m_N^{-2}$, but by the W-boson pair production $\sigma(\bar{N}N \to W^-W^+) \propto m_N^2$. They concluded [5] that "there is no cosmological upper bound on the masses of very heavy stable neutrinos". They assumed the standard weak coupling of (Dirac) neutrinos.

However, Griest and Kamionkowski [6] successfully put an upper bound on the mass of the CDM particle estimating the upper bound of the total annihilation cross section without reference to any particular interaction model or theory.

Nevertheless, it should be remembered that heavy neutrinos have a substantially different scaling of the cross sections in the fermion or boson pair production. If one excludes the W^-W^+ production, one can reach a cosmologically acceptable abundance of a heavy neutrino as a dark matter particle with a mass $m_N \simeq 900GeV$ [5].

It is now necessary to define our framework for a study of heavy neutrinos. We work within an electroweak theory proposed some years ago [7]. We called it (BY) theory in that paper. It differs from the Standard Model(SM) (called (AX) theory) in two essential ingredientes: (1) Nambu-Goldstone scalars carry nonvanishing lepton numbers and only Majorana fields acquire masses at tree level, (2) there is no Higgs scalar and the principle of the noncontractible space (the existence of finite UV scale) is introduced into the local gauge field theory as a symmetry-breaking mechanism, fixing the tree-level weak boson and Majorana fermion masses. The theory contains three light and three heavy Majorana neutrinos. Further insights and discussions could be found in Ref. [7]. Here we concentrate on the leptonic sector.

Let us recall the structure of the four-component Dirac spinor:

$$\Psi_D \equiv \begin{pmatrix} \eta_a \\ \dot{\xi}^{\dot{a}} \end{pmatrix}, \tag{1}$$

$$\eta_a \; transforms \; under \; a \; matrix \; S \; of \; SL(2,C),$$

$$\dot{\xi}^{\dot{a}} \; transforms \; under \; a \; matrix \; (S^{-1})^* \; of \; SL(2,C).$$

To retain all degrees of freedom, one can choose the following set of fermion fields:

$$e_L, \; e_R, \; \Psi_L, \; (\Psi^C)_L. \tag{2}$$

These fields interact with $SU(2)\times U(1)$ gauge fields and Nambu-Goldstone scalars as follows:

$$\mathcal{L} \;=\; \mathcal{L}_{lep} + \mathcal{L}_{g.bos} + \mathcal{L}_{scal} + \mathcal{L}^M_{Yuk} + \mathcal{L}_{g.fix} + \mathcal{L}_{FP}, \tag{3}$$

$$\begin{aligned}
&\mathcal{L}_{lep} = \bar{R} i\gamma^\mu(\partial_\mu + ig' B_\mu)R + \bar{L} i\gamma^\mu(\partial_\mu + \tfrac{i}{2}g' B_\mu - ig\tfrac{\tau^i}{2}A^i_\mu)L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R,\\
&\mathcal{L}_{g.bos} = -\tfrac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu},\\
&\mathcal{L}_{scal} = (\partial_\mu \Phi^\dagger + i\tfrac{g'}{2}B_\mu\Phi^\dagger + i\tfrac{g}{2}\tau^i A^i_\mu \Phi^\dagger)(\partial^\mu\Phi - i\tfrac{g'}{2}B^\mu\Phi - i\tfrac{g}{2}\tau^i A^{i\mu}\Phi),\\
&\mathcal{L}^M_{Yuk} = -Y^\psi_M \bar{L^C}\Phi\psi_R + h.c.,\\
&definitions: \; e = charged \; lepton, \; \psi = neutral \; Dirac \; lepton,\\
&L = \begin{pmatrix} \psi_L \\ e_L \end{pmatrix}, \; R = e_R;\\
&F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^{ijk}A^j_\mu A^k_\nu, \; B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu;\\
&\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \; L^C = \begin{pmatrix} (e^C)_L \\ (\Psi^C)_L \end{pmatrix},\\
&\phi^0(x) \;=\; (v + i\chi^0(x))/\sqrt{2}, \; v = symmetry \; breaking \; parameter,\\
&\phi^\pm, \chi^0 = Nambu - Goldstone \; scalars.
\end{aligned}$$

The physical spectrum of neutral leptons looks like [7, 8]

$$\begin{pmatrix} m_L & m_D \\ m_D & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix},$$

$$a^2 + b^2 = 1,$$

$$\Psi_N = a_N f + b_N F,\ \Psi_{\nu'} = a_{\nu'} f + b_{\nu'} F, \tag{4}$$

$$f \equiv \frac{1}{\sqrt{2}}(\Psi_L + (\Psi_L)^C),\ F \equiv \frac{1}{\sqrt{2}}(\Psi_R + (\Psi_R)^C),$$

$$\lambda_N = \frac{1}{2}(m_L + \sqrt{m_L^2 + 4m_D^2}),\ \lambda_{\nu'} = \frac{1}{2}(m_L - \sqrt{m_L^2 + 4m_D^2}), \tag{5}$$

$$a_N = \frac{m_L + \sqrt{m_L^2 + 4m_D^2}}{(8m_D^2 + 2m_L^2 + 2m_L\sqrt{m_L^2 + 4m_D^2})^{\frac{1}{2}}},\ b_N = \frac{2m_D a_N}{m_L + \sqrt{m_L^2 + 4m_D^2}},$$

$$a_{\nu'} = \frac{\sqrt{m_L^2 + 4m_D^2} - m_L}{(8m_D^2 + 2m_L^2 - 2m_L\sqrt{m_L^2 + 4m_D^2})^{\frac{1}{2}}},\ b_{\nu'} = \frac{2m_D a_{\nu'}}{m_L - \sqrt{m_L^2 + 4m_D^2}},$$

$$m_L \gg m_D \Rightarrow \lambda_N \simeq m_L,\ \lambda_{\nu'} \simeq -\frac{m_D^2}{m_L},$$

$$a_N \simeq 1,\ b_N \simeq \frac{m_D}{m_L},\ a_{\nu'} \simeq \frac{m_D}{m_L},\ b_{\nu'} \simeq -1,$$

$$\Psi_\nu \equiv \gamma_5 \Psi_{\nu'} \Rightarrow \lambda_\nu = -\lambda_{\nu'}. \tag{6}$$

The lightest heavy neutrino has the mass $m_{N_e} \simeq 0.5 TeV$, but heavier Majorana particles (N_μ and N_τ) acquire masses predominantly from the quantum loops with a Nambu-Goldstone scalar in the strong coupling regime of Dyson-Schwinger equations [7]. However, the effective strong coupling for fermions with masses of a few TeV is saturated in bootstrap equations and its value (like the QCD strong coupling in the infrared domain) is not so high. The cross section for heavy neutrinos annihilating into a heavy neutrino pair, calculated in the 't Hooft-Feynman gauge is

$$
\begin{aligned}
\sigma_{\chi^0}(N_i N_i \to N_j N_j) &= \frac{\pi}{\sin^4\theta_W}(\tilde{\alpha_e})_i(\tilde{\alpha_e})_j \\
&\times \frac{1}{(s-m_{\chi^0}^2)^2+\Gamma_{\chi^0}^2 m_{\chi^0}^2}\frac{s-2m_{N_j}^2}{s}\sqrt{(s-4m_{N_i}^2)(s-4m_{N_j}^2)}.
\end{aligned}
\tag{7}
$$

Notice that we cannot decouple Nambu-Goldstone scalars when they are in the strong coupling regime with fermions. Only a complete solution of the whole set of Dyson-Schwinger equations can give us gauge-invariant observables. Thus, the preceding cross section formula should be read off as only for effective quantities. Because of the see-saw suppression, the cross sections mediated through electroweak gauge bosons are negligible and, in addition, there is no W^-W^+ pair production via the χ^0 Nambu-Goldstone boson because $Vertex(\chi^0 W^- W^+) \equiv 0$ and there is no Higgs scalar in the theory [7]. The scaling of the cross section in Eq.(7) is the same as that of the cross section $\sigma_Z(\bar{N}N \to \bar{f}f)$ (Eq.(2) of Ref. [5]): $\sigma_{\chi^0} \propto m_N^{-2}$.

Knowing the predominant contribution to the total annihilation cross section, we can estimate the abundances of heavy neutrinos. The freeze-out temperature of the CDM particle depends on the cross section only logarithmically [2], so it is not neccessary to solve the freeze-out condition [5] for σ_{χ^0}. We can compare cross sections and make the following estimates:

$$s \gg m_{N_j}^2, m_{\chi^0}^2, \Gamma_{\chi^0}^2,$$

$$\Rightarrow \sigma_{\chi^0} \simeq \frac{\pi}{\sin^4\theta_W}(\tilde{\alpha_e})_i(\tilde{\alpha_e})_j \beta_{N_i}\frac{1}{s},$$

$$\beta_N = (1-4m_N^2/s)^{1/2},\ s \simeq 4m_N^2 + 6m_N T_f,\ T_f \sim \frac{m_N}{30},$$

$$x_f \equiv \frac{m_N}{T_f},\ x_f \simeq 17\ (m_N \simeq 2GeV),\ x_f \simeq 25\ (m_N \geq 1TeV),$$

$$(\tilde{\alpha_e})_i \sim 0.5 \Rightarrow \sigma_{\chi^0} \simeq \frac{1}{m_{N_i}^2},$$

$$\Omega_N \propto \frac{(n+1)x_f^{n+1}}{\sigma_0}, \ \langle \sigma \mid v \mid \rangle \equiv \sigma_0 x^{-n},$$

$$\sigma_Z(m_N \simeq 0.9TeV) \simeq \sigma_{\chi^0}(m_{N_i}), \ \Omega_N \simeq 1$$

$$\Rightarrow m_{N_i} = \mathcal{O}(10TeV). \tag{8}$$

Although we are limited with the knowledge of the effective coupling of fermions to the Nambu-Goldstone scalars, one can conclude that heavy Majorana neutrinos with a mass of order $\mathcal{O}(10TeV)$ are good candidates for the CDM particle.

If one assumes that the radiative decay of the CDM particle dominates, we can estimate its lifetime from the measurements of the differential energy flux of the diffuse photon background [2, 3]:

$$\Omega_N \propto m_N n_N, \ \Omega_N \simeq 1$$

$$\Rightarrow n_N \propto m_N^{-1},$$

$$\frac{d\mathcal{F}}{dEd\Omega} \propto \frac{n_N}{\tau_N} \propto \frac{1}{m_N \tau_N}, \tag{9}$$

$$\tau_N = \mathcal{O}(10^{23}s) \ (m_N = \mathcal{O}(1GeV)) \Rightarrow \tau_N = \mathcal{O}(10^{19}s) \ (m_N = \mathcal{O}(10TeV)).$$

Note that the estimate of the lifetime depends strongly only on the mass of the CDM particle(the cross section is constrained by the cosmic CDM abundance condition). The present estimates of the mass and lifetime of the CDM particle are further challenges for a theory, so the next section is devoted to a study of its lifetime.

3 Lifetime of heavy neutrinos

Searching for the tree-level decay processes of neutrinos, one can recall that the tree-level decays of light neutrinos are kinematically forbidden. Heavy neutrinos couple to electroweak gauge bosons with see-saw suppression factors, but in our theory these factors vanish at tree level [7] because $m_{D_i}(tree\ level) \equiv 0$, i=flavour qauntum number. Just like for light neutrinos[3], we have to calculate the two-body radiative flavour changing decay of heavy neutrinos at the quantum loop order.

Let us be concerned with the $N_i \to \nu_j \gamma$ decay via the charged weak current loop. When dealing with processes containing light and heavy neutrinos, one has to symmetrize fermion states with respect to the following interchange of fields: $\psi_i \leftrightarrow \psi_i^c \Rightarrow f_i \leftrightarrow F_i,\ i = flavour$. Nature should not recognize what we define as a particle or an antiparticle. Acknowledging the see-saw mixing coefficientes from Eq.(4) and after the particle-antiparticle symmetrization, one can conclude

$$\Gamma(N_i \to \nu_j \gamma) \propto a_{N_i} a_{\nu'_j} + b_{\nu'_i} b_{N_j},\ \forall\ i, j,$$

$$a_{N_i} = -b_{\nu'_i}\ and\ a_{\nu'_j} = b_{N_j} \Rightarrow \Gamma(N_i \to \nu_j \gamma) \equiv 0. \tag{10}$$

We shall now study the decay processes $N_i \to N_j \gamma$ induced by the loop exchange of the W weak boson and charged leptons. The effective transition operator will be estimated preturbatively, so one has to choose the renormalization conditions for the flavour mixing of leptons.

The usual wisdom for fermion mixing is to apply the on-shell fermion mixing scheme of Ref.[9]. It has been shown that in flavour-changing lepton radiative processes, this scheme causes the appearance of the dimension-four operators [10] besides the standard dimension five-operators [3]. However, the authors of Ref.[11] have recently shown that the on-shell renormalization scheme is not consistent because it violates Ward-Takahashi identities and leads to gauge-dependent physical amplitudes. Instead, one can introduce a natural and consistent prescription where flavour-changing self-energies vanish at zero momentum[11]. From the Ward-Takahashi identity and the conservation of the electromagnetic current one can conclude that new renormalization conditions of Ref.[11] do not induce the appearance of four-dimensional operators in flavour changing lepton radiative decays.

The effective see-saw suppressed transition operator for $N_i \to N_j\gamma$, $(i,j = flavours)$ is evaluated in the t'Hooft-Feynman gauge (loops with exchanged WWl and Wll, l=charged lepton) because only a strongly coupled heavy neutrino-Nambu-Goldstone scalar (decoupled in the unitary gauge) system can produce the mass splitting of heavy neutrinos:

$$\mathcal{A}_\mu = [H(0) + H_5(0)\gamma_5]\sigma_{\mu\rho}(p_2)^\rho, \tag{11}$$

$$\begin{aligned}
H &= \frac{eg^2}{16\pi^2}\sqrt{\frac{m_{\nu_i}m_{\nu_j}}{m_{N_i}m_{N_j}}}\sum_l U^*_{lj}U_{li}(H_L + H_R),\\
H_5 &= \frac{eg^2}{16\pi^2}\sqrt{\frac{m_{\nu_i}m_{\nu_j}}{m_{N_i}m_{N_j}}}\sum_l U^*_{lj}U_{li}(H_R - H_L),\\
H_L &= m_{N_j}(C_0 + C_{11} + C_{12} + C_{23}),\\
H_R &= m_{N_i}(-C_0 + C_{12} + \tilde{C}_{12} - 2C_{11} - \frac{1}{2}\tilde{C}_{11} - C_{21} + C_{23}),\\
C_0 &= C_0(p_1^2, p_2^2, p_3^2; M_W, m_l, m_l),\\
\tilde{C}_0 &= C_0(p_1^2, p_2^2, p_3^2; m_l, M_W, M_W),\\
p_1^2 &= m^2_{N_i},\ p_2^2 = 0,\ p_3^2 = m^2_{N_j}, m_l = charged\ lepton\ mass,\\
& \quad for\ further\ definitions\ see\ Appendix\ A.
\end{aligned}$$

The amplitude is ultraviolet(UV) finite, but it contains infrared(IR) singularity in the limes $m_l \to 0$. It can be visualized that the IR singularity comes from the following two Green's functions:

$$\begin{aligned}
\Re B_0(0; m_l, m_l) &= -\ln m_l^2 + ...,\\
\Re C_0 &= \frac{1}{p_1^2 - p_3^2}(\frac{1}{2}\ln^2 w_1 - \frac{1}{2}\ln^2 w_3) + ...,\\
w(p^2, M_W, m_l) &= \frac{1}{2p^2}(p^2 - M_W^2 + m_l^2 - \sqrt{(p^2 - M_W^2 + m_l^2)^2 - 4p^2m_l^2}),\\
w_1 &= w(p_1^2, M_W, m_l),\ w_3 = w(p_3^2, M_W, m_l).
\end{aligned}$$

The IR singularity can be removed in a natural way through the leptonic GIM mechanism:

$$H = \lim_{M \to 0} \sum_l [\mathcal{H}(m_l^2, ...) - \mathcal{H}(m_l^2 = M^2, ...)],$$
$$similarly\ for\ H_5.$$

The IR singular and constant terms of the whole amplitude are now subtracted away.

Instead of the Higgs mechanism, the finite UV scale is introduced into the theory, so one has to study the finite-scale effects. They enter into the calculations through the evaluations of scalar Green's functions [12]. One can naively expect large corrections, but the explicit evaluation (see Appendix B) tells us that they are three orders of magnitude smaller, thus much smaller than uncertainties in masses. We shall neglect this effect in the numerical estimates of lifetimes. The reason for small corrections is that for these decay processes we need the knowledge of the Green's functions in the timelike region for very high masses ($m_{N_i} \gg \Lambda$). The contribution from the timelike region dominates over that of the spacelike region. On the contrary, in QCD one studies the spacelike region with a cutoff Λ, thus a large deviation will be encountered [12] for the scale $\mu \geq \Lambda$.

A straightforward calculation gives the partial decay width of the Majorana heavy neutrino(assumed the same CP eigenvalues of neutrinos [3]):

$$\Gamma(N_i \to N_j \gamma) = \frac{(m_{N_i}^2 - m_{N_j}^2)^3}{8\pi m_{N_i}^3} \mid H_5 \mid^2 . \tag{12}$$

Because of the fact that $m_{N_i}^2 \gg M_W^2$, a partial decay width is insensitive to the weak boson mass to the leading order:

$$\Gamma(N_i \to N_j \gamma) \propto m_{N_i} \sin^2(2\theta_{ij}) (\frac{m_{\nu_i} m_{\nu_j}}{m_{N_i} m_{N_j}}) (\frac{m_{l_i}}{m_{N_j}})^4. \tag{13}$$

Owing to this scaling, the gauge dependence of the effective transition operator is of subleading order and there is no need to study the gauge cancellations in detail. The heavy-quark symmetry of Isgur and Wise or the Appelquist-Carazzone decoupling theorem are also examples in field theory where a good approximation in a certain sector of the complete theory describes the actual physical situation correctly.

Next section is devoted to final numerical evaluations and physical conclusions.

4 Results and conclusions

From the requirement for the abundance of the CDM particle and the requirement of the cosmic diffuse photon background we extracted the information about possible masses and lifetimes of heavy neutrinos. The general scaling of the lifetime on light- and heavy-neutrino massees and mixing angles suggests that there is no dependence on the mass of the decaying particle, because of the cancellation(see Eq.(13)).

We present our numerical results with the assumption that in each decay channel only the mixing between two flavours is allowed (for example, in $N_2 \to N_1\gamma$ only θ_{12} does not vanish). This is done to gain the transparency of the presented results. The same observation is valid for the scaling relation in Eq.(13).

$$m_{N_e} = 0.5TeV,\ m_{N_\mu} = 10TeV,\ m_{N_\tau} = 100TeV,$$

$$m_{\nu_e} = 0.05eV,\ m_{\nu_\mu} = 0.5eV,\ m_{\nu_\tau} = 5eV,$$

$$\theta = 0.2\ (flavour\ mixing\ angle),$$

$$\tau = [\sum_a \tau_a^{-1}]^{-1},$$

$$\tau(N_\tau \to N_\mu\gamma) = 6.13 \cdot 10^{18}s,\ \tau(N_\tau \to N_e\gamma) = 3.80 \cdot 10^{15}s,$$

$$\tau(N_\mu \to N_e\gamma) = 1.78 \cdot 10^{19}s. \tag{14}$$

The lack of our knowledge of the mixing angles and neutrino masses does not prevent us from drawing main conclusions of this paper concerning heavy neutrinos:

(1) Acknowledging the see-saw relation for light-neutrino masses and the relation for the mixing angles $\theta_W = 2(\theta_{12} + \theta_{23} + \theta_{31})$ [7], the (BY) theory predicts that the heavy neutrino N_μ with the mass $\mathcal{O}(10TeV)$ and the lifetime $\tau = \mathcal{O}(10^{19})s$ is a CDM particle which can solve the problem of the cosmic diffuse photon background; this means that a further detailed astronomical study of the diffuse photon background could be useful for the study of the dynamics of the CDM particle, which could be supplemented by terrestrial measurements (for example, the DAMA experiment at the Gran Sasso National Laboratory); it is not excluded that processes with heavy neutrinos could explain the recently observed(EGRET) diffuse gamma halo around our Galaxy.
(2) Essential ingredients of the (BY) theory, which gives successful phenomenological answers, are the peculiar chiral structure of the theory, the absence of the Higgs scalar and the presence of the finite UV scale to fix the scale of heavy neutrinos and weak gauge bosons, as well as the presence of the UV finite self-consistent bootstrap system of equations leading to the finite number of fermion families.
(3) The heavy-neutrino N_τ as a cosmologically unstable particle $\tau_{N_\tau} < H_0^{-1}$ can also contribute to the cosmic diffuse photon background if it decays after recombination [2].
(4) The forthcoming higher-luminosity measurements at Tevatron could discovere the N_e heavy neutrino ($m_{N_e} \simeq 0.5TeV$) and confirm the nonresonant enhancement of the QCD amplitudes beginning in the vicinity of Λ ($\lim_{\mu\to\infty} \alpha_s^\Lambda(\mu) \neq 0$) [12]. Also note the unsettled problem of the anomalous b-quark weak coupling (LEP and SLD) as a possible consequence of the finite scale effect in the loop corrections with the t-quark exchange [12].
(5) The Einstein-Cartan cosmology requires that $\Omega_m \simeq 2$, thus the CDM particle must be the spinning particle (fulfilled for the (BY) theory) [13]; this setting of the average mass density of the Universe is a consequence of the following bootstrap at the level of the Einstein-Cartan equations: the number density of the particle that dominates the energy momentum density, at the same time dominates the spin density of the Universe, but the energy momentum and spin of matter are coupled to the curvature and torsion of spacetime with the same coupling constant.
(6) The current controversies [14] concerning the average mass density of the Universe could be resolved only with the most general cosmological model that contains cold and hot dark matter, cosmological constant, baryons and

CMBR and the metric not only with expansion, but also with a small amount of vorticity, acceleration and shear [13]; namely, one can solve the primordial mass density fluctuation within the Einstein-Cartan cosmology only beyond the standard Friedmann-Lemaître-Robertson-Walker metric [15]; it was shown a long time ago that the primordial vorticity could generate a cosmic magnetic field [16].

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Appendix A

Here we display the scalar and tensor Green's functions used in the evaluation of the effective transition flavour-changing operator:

$$A(m) = \frac{1}{\imath\pi^2}\int d^4q \frac{1}{q^2 - m^2 + \imath\varepsilon},$$

$$B_0(p^2; m_1, m_2) = \frac{1}{\imath\pi^2}\int d^4q \frac{1}{(q^2 - m_1^2 + \imath\varepsilon)((q+p)^2 - m_2^2 + \imath\varepsilon)},$$

$$p_\mu B_1(p^2; m_1, m_2) = \frac{1}{\imath\pi^2}\int d^4q \frac{q_\mu}{(q^2 - m_1^2 + \imath\varepsilon)((q+p)^2 - m_2^2 + \imath\varepsilon)},$$

$$g_{\mu\nu}B_{22} + p_\mu p_\nu B_{21} = \frac{1}{\imath\pi^2}\int d^4q \frac{q_\mu q_\nu}{(q^2 - m_1^2 + \imath\varepsilon)((q+p)^2 - m_2^2 + \imath\varepsilon)},$$

$$p_1 + p_2 + p_3 = 0,$$

$$C_0(p_1^2, p_2^2, p_3^2; m_1, m_2, m_3) = \frac{1}{\imath\pi^2}\int \frac{d^4q}{(q^2 - m_1^2 + \imath\varepsilon)((q+p_1)^2 - m_2^2 + \imath\varepsilon)((q-p_3)^2 - m_3^2 + \imath\varepsilon)},$$

$$p_1^\mu C_{11} + p_2^\mu C_{12} = \frac{1}{\imath\pi^2}\int d^4q \frac{q^\mu}{(q^2 - m_1^2 + \imath\varepsilon)((q+p_1)^2 - m_2^2 + \imath\varepsilon)((q-p_3)^2 + m_3^2 + \imath\varepsilon)},$$

$$g^{\mu\nu}C_{24} + p_1^\mu p_1^\nu C_{21} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu)C_{23}$$
$$= \frac{1}{\imath\pi^2}\int d^4q \frac{q^\mu q^\nu}{(q^2 - m_1^2 + \imath\varepsilon)((q+p_1)^2 - m_2^2 + \imath\varepsilon)((q-p_3)^2 + m_3^2 + \imath\varepsilon)},$$

$$B_1(p^2; m_1, m_2) = \frac{1}{2p^2}[A(m_1) - A(m_2) + (m_2^2 - m_1^2 - p^2)B_0(p^2; m_1, m_2)],$$
$$B_{22}(p^2; m_1, m_2) = \frac{1}{6}[A(m_2) + 2m_1^2 B_0(p^2; m_1, m_2) + (p^2 + m_1^2 - m_2^2)B_1(p^2; m_1, m_2)],$$
$$B_{21}(p^2; m_1, m_2) = \frac{1}{3p^2}[A(m_2) - m_1^2 B_0(p^2; m_1, m_2) - 2(p^2 + m_1^2 - m_2^2)B_1(p^2; m_1, m_2)],$$

$$C_{12} = \frac{1}{(p_1\cdot p_2)^2 - p_1^2 p_2^2}[p_1^2\Sigma_2 + (p_1^2 + p_1\cdot p_2)\Sigma_1],$$

$$C_{11} = \frac{1}{p_1^2}(\Sigma_1 - p_1\cdot p_2 C_{12}),$$

$$\Sigma_1 = \frac{1}{2}B_0(p_3^2; m_1, m_3) + \frac{1}{2}(-p_1^2 - m_1^2 + m_2^2)C_0(p_1^2, p_2^2, p_3^2; m_1, m_2, m_3) - \frac{1}{2}B_0(p_2^2; m_2, m_3),$$
$$\Sigma_2 = -\frac{1}{2}B_0(p_1^2; m_1, m_2) + \frac{1}{2}(p_3^2 - m_3^2 + m_1^2)C_0(p_1^2, p_2^2, p_3^2; m_1, m_2, m_3) + \frac{1}{2}B_0(p_2^2; m_2, m_3),$$

$$C_{23} = \frac{1}{2(p_1^2 p_2^2 - (p_1\cdot p_2)^2)}[(-3p_1^2 - 4p_1\cdot p_2)\Lambda_2 - 3p_1^2\Lambda_4 + p_1\cdot p_2\Lambda_1 - p_2^2\Lambda_3],$$

$$C_{21} = -\frac{1}{p_1\cdot p_2}[\Lambda_2 + \Lambda_4 + p_2^2 C_{23}],$$

$$C_{22} = \frac{1}{p_1\cdot p_2}[\Lambda_3 - p_1^2 C_{23}],$$

$$C_{24} = \Lambda_2 - p_1^2 C_{21} - p_1 \cdot p_2 C_{23},$$

$$\Lambda_1 = B_0(p_2^2; m_2, m_3) + m_1^2 C_0,$$

$$\Lambda_2 = \frac{1}{2} B_1(p_3^2; m_1, m_3) + \frac{1}{2}(-p_1^2 + m_2^2 - m_1^2) C_{11},$$

$$\Lambda_3 = \frac{1}{2} B_1(p_3^2; m_1, m_3) - \frac{1}{2} B_1(p_2^2; m_2, m_3) + \frac{1}{2}(-p_1^2 + m_2^2 - m_1^2) C_{12},$$

$$\Lambda_4 = -\frac{1}{2} B_1(p_1^2; m_1, m_2) + \frac{1}{2}(p_3^2 - m_3^2 + m_1^2) C_{11}.$$

Appendix B

The real parts of the two- and three-point scalar Green's functions in the noncontractible space are given as in Ref. [12]:

$$\Re B_0^\Lambda(p^2; m_1, m_2) = \left(\int_0^{\Lambda^2} dy K(p^2, y) + \theta(p^2 - m_2^2) \int_{-(\sqrt{p^2} - m_2)^2}^0 dy \Delta K(p^2, y)\right) \frac{1}{y + m_1^2},$$

$$K(p^2, y) = \frac{2y}{-p^2 + y + m_2^2 + \sqrt{(-p^2 + y + m_2^2)^2 + 4p^2 y}},$$

$$\Delta K(p^2, y) = \frac{\sqrt{(-p^2 + y + m_2^2)^2 + 4p^2 y}}{p^2}.$$

The integration in the second term is performed from the branch point of the square root $\sqrt{(-p^2 + y + m_2^2)^2 + 4p^2 y} \equiv \imath Z$ and the additional kernel is derived as the difference: $\Delta K(p^2, y) = K(p^2, y) - K^*(p^2, y) = \frac{2y}{-p^2 + y + m_2^2 + \imath Z} - \frac{2y}{-p^2 + y + m_2^2 - \imath Z}$.

The integration over singularities is supposed to be the principal-value integration.

In the case of the two-point Green's function B_0^Λ, we need the explicit form of the additional term for the integration in the timelike region because the integration in the spacelike region is divergent in the limes $\Lambda \to \infty$. However, the three-point scalar Green's functions are UV-convergent and we do not need to know the explicit form of the additional terms because they do not depend on the UV cut-off and we can use the analytical continuation of the standard Green's functions written in terms of the dilogarithms[12, 17]:

$$\Re C_0^\Lambda(p_i, m_j) = \int_0^{\Lambda^2} dq^2 \Phi(q^2, p_i, m_j) + \int_{TD} dq^2 \Xi(q^2, p_i, m_j),$$
$$\Re C_0^\Lambda(p_i, m_j) = ReC_0^\infty(p_i, m_j) - \int_{\Lambda^2}^\infty dq^2 \Phi(q^2, p_i, m_j),$$
$$\Phi \equiv function\ derived\ by\ the\ angular\ integration\ after\ Wick's\ rotation,$$
$$C_0^\infty \equiv standard\ 't\ Hooft - Veltman\ scalar\ function,$$
$$TD \equiv timelike\ domain\ of\ integration.$$

This equation is valid for arbitrary external momenta. The same formula is applicable to the higher n-point one loop scalar Green's functions.

To confirm the claim that the finite scale effects in the heavy-neutrino radiative decay are small, we evalute the $C \equiv \tilde{C}_0$ Green's function as an example:

$$\Re C^\Lambda = \Re C^\infty - \triangle\Gamma,$$

$$\triangle\Gamma = \Gamma_\infty - \Gamma_\Lambda,$$

$$\Gamma_\Lambda = -\frac{2}{\pi}\int_0^\Lambda dq \frac{q^3}{q^2 + M_W^2}(I_1 + I_2),$$

$$I_1 = \int_{-1}^{+1} dx \frac{1}{(q^2 + m_1^2 + m_l^2)^2 + 4m_1^2 q^2 x^2} \frac{-m_1 \mid x \mid}{k}$$

$$\times(\arctan \frac{q^2 + m_2^2 + m_l^2 + 2kq\sqrt{1-x^2}}{2(m_1 + k)q \mid x \mid} - \arctan \frac{q^2 + m_2^2 + m_l^2 - 2kq\sqrt{1-x^2}}{2(m_1 + k)q \mid x \mid}),$$

$$I_2=\int_{-1}^{+1}dx\frac{1}{(q^2+m_1^2+m_l^2)^2+4m_1^2q^2x^2}\frac{q^2+m_1^2+m_l^2}{4kq}$$

$$\times\ln\frac{(q^2+m_2^2+m_l^2+2kq\sqrt{1-x^2})^2+4(m_1+k)^2q^2x^2}{(q^2+m_2^2+m_l^2-2kq\sqrt{1-x^2})^2+4(m_1+k)^2q^2x^2},$$

$$k\equiv\frac{m_2^2-m_1^2}{2m_1},$$

$$p_1^2=m_1^2=(10TeV)^2,\ p_3^2=m_2^2=(0.5TeV)^2,\ \Lambda=326GeV,$$

$$\triangle\Re C^\infty\equiv\Re C^\infty(m_l=m_\mu)-\Re C^\infty(m_l=m_e)=1.00\cdot10^{-8}TeV^{-2},$$

$$\triangle(\triangle\Gamma)\equiv\triangle\Gamma(m_l=m_\mu)-\triangle\Gamma(m_l=m_e)=-6.87\cdot10^{-12}TeV^{-2}$$

$$\Rightarrow\mid\triangle\Re C^\infty\mid\gg\mid\triangle(\triangle\Gamma)\mid.$$

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